π RE-COUNT and RECALL

Paul Scott
Adelaide, SA
mail@paulscott.info

The number π

This year we have been investigating the number $\pi \approx 3.14159$, defined to be the ratio C/d of the circumference C to the diameter d of any given circle.

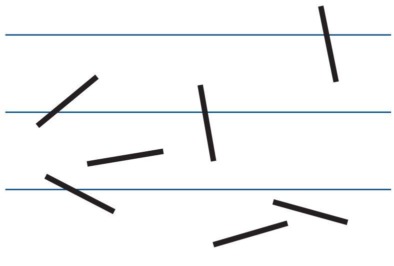
Here we look at another of those surprising and unexpected places where π occurs, and then think about some ways of remembering all those digits in the expansion of π.

Re-count

Count Buffon, or more strictly, Georges-Louis Leclerc, Comte de Buffon, lived from 1707 to 1788. He was a French naturalist, mathematician, biologist, cosmologist and author. In mathematics he is remembered for an experiment in probability — the Buffon needle experiment.

Class investigation

1. Trim a matchstick to a length of exactly 3 cm. Now take a large sheet of drawing paper, and rule a series of parallel lines 3 cm apart right across it. Repeatedly toss the match onto the paper, and note whether it crosses (or touches) any of the ruled lines.



Complete the table below.
Discuss the behaviour of N_1/N_2 as N_2 gets large.

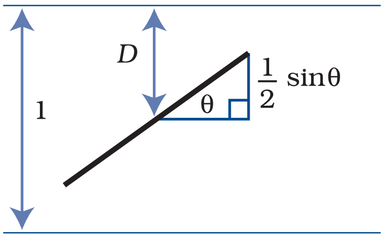
Note: A large number of tosses is required for a worthwhile result. The experiment can be shortened by
(a) tossing ten matches at a time;
(b) having several groups do the experiment and pooling the results.

Number of crossings (N_1)					
Number of tosses (N_2)	200	400	600	800	1000
Ratio N_1/N_2 (as a decimal)					

Rather surprisingly, the ratio N_1/N_2 approaches the value $2/\pi$ (≈ 0.63662) as N_2 gets large. That is, the probability of a match crossing (or touching) any of the lines is $2/\pi$; or, we can write $\pi \approx \dfrac{2N_2}{N_1}$

Why π appears

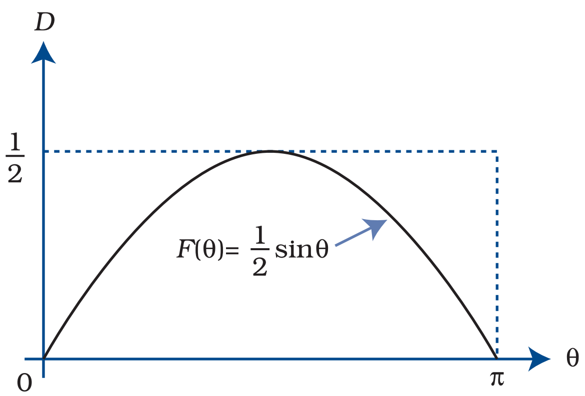
At first sight it is surprising to see the number π appearing here.



Let us take the length of the match to be one unit and the distance between the lines to be also one unit. There are two variables, the angle theta (θ) at which the match falls and the distance (D) from the centre of the match to the closest line. We may assume that $0 \le \theta \le 180°$ and that the angle is measured from a line parallel to the lines on the paper. The distance from the centre to the closest line can never be more than half the distance between the lines. The graph below depicts this situation.

The match in the picture above misses the lines. The match will hit a line if the closest distance to a line (D) is less than or equal to $\frac{1}{2}$ times the sine of theta; that is, $D \le \frac{1}{2}.\sin\theta$. How often will this occur?

In the graph below, we plot D up the y-axis and θ along the x-axis. The values on or below the curve $F(\theta) = \frac{1}{2}.\sin\theta$ represent a hit ($D \le \frac{1}{2}.\sin\theta$). Thus, the probability of a success is the ratio of the shaded area to the area of the entire rectangle. What is this value?



The area of the shaded portion is found by integrating $\frac{1}{2}.\sin\,\theta$ between 0 and π. It is easily found that this area is 1. The area of the entire rectangle is $\pi/2$. So, the probability of a hit is:

$$\frac{1}{\left(\frac{\pi}{2}\right)} \text{ or } \frac{2}{\pi}$$

that is, approximately 0.6366197.

We observe that the occurrence of π in this result is essentially due to the occurrence of the sine function in the calculation.

Further investigations

1. Buffon's result can be generalised by allowing the distance between the lines to be different from the length of the match (or needle in the original). Investigate this.

2. Could we obtain a similar result in one dimension along a line? For example, a straight line has dots marked at 6 cm intervals. Suppose we can devise a means of dropping a 3 cm matchstick along the line. What is the probability that the match will cross or touch one of the dots? [This simplified form of Buffon's Problem is really quite easy. Consider different positions of the match, and decide where the left hand endpoint of the match must be for a dot to be covered.]

3. Instead of carrying out Buffon's experiment with a matchstick (line segment), we might decide to use other plane geometrical figures: for example a cardboard triangle or square. Investigate this. See the chapter 'The Regularity of Randomness' in the book *Exploring Mathematical Thought* by S. J. Taylor, (Ginn & Co., 1970) for an interesting account of this.

Recall

"Thirty days hath September…"
"Every Good Boy Deserves Fruit"

Rhymes and sayings like these are often used for bringing certain facts to mind. Such memory aids are often called *mnemonics* after the Greek goddess of memory, Mnemosyne.

Many mnemonics have been devised to help lesser mortals remember the first digits in the decimal expansion of π.

$\pi = 3.141\,592\,653\,589\,793\,238\,462\,643\,383\,279\ldots$

In these mnemonics, the number of letters in successive words gives the digits in the expansion. For example, the first eight figures can be obtained from:

May I have a large container of coffee?

Another mnemonic, due to Sir James Jeans, is:

How I want a drink, alcoholic of course, after the heavy chapters involving quantum mechanics.

Of course, the coffee is probably better for you, even after quantum mechanics! The following are rather more ambitious mnemonics in verse:

Que j'aime à faire apprendre
Un nombre utile aux sages!
Immortel Archimède, artiste, ingénieur,
Qui de ton jugement peut priser la valeur!
Pour moi ton problème
Eut de pareils avantages.

Now I — even I — would celebrate
In rhymes inept the great
Immortal Syracusen rivalled nevermore,
Who by his wondrous lore,
Untold us before,
Made the way straight
How to circles mensurate.

Although initial attempts to calculate π were slow and laborious, these days with the use of high speed computers, the decimal expansion of π is known to thousands of places. In 1958, Gazis and Herman proposed a short "history" of these calculations which happens to give the decimal expansion of π to 40 figures. Notice that the hyphen represents zero.

All I know I could disregard as hardly worth our while relating. Thousands laboured computing for pi but obtained very little. In modern days one can increase the pi figures utilising built-up electric monsters. What a marvellous science!

For further investigation

1. Complete the following mnemonic for π:

   Yes, I want a juicy hamburger or French apple ___ .

2. Try making up your own mnemonic for π.

Bibliography

Scott, P. R.. (1974). *Discovering the Mysterious Numbers*. Cheshire.

Pedoe, D. (1973). *The Gentle Art of Mathematics*. Pelican. [see the chapter 'Chance and Choice' for a good introduction to Buffon's problem]

http://www.mste.uiuc.edu/reese/buffon/buffon.html [an online discussion and simulation of Buffon's Needle experiment]

http://mathworld.wolfram.com/PiWordplay.html [A large collection of mnemonics for pi]